Cambridge O Level

Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Partial Marks
1	$5x + 3 = 3x - 1$ oe or $5x + 3 = 1 - 3x$ oe	**M1**	
	$x = -2$ and $x = -0.25$ only mark final answer	**A2**	nfww **A1** for $x = -2$ ignoring extras implies M1 if no extras seen If M0 then **SC1** for any correct value with at most one extra value
	Alternative method $(5x+3)^2 = (1-3x)^2$ oe soi	**M1**	
	$16x^2 + 36x + 8 = 0$ oe	**A1**	
	$x = -0.25$, $x = -2$ only; mark final answer	**A1**	
2	**Without using a calculator…** Sufficient evidence must be seen to be convinced that a calculator has not been used. Withhold the mark for any step that is unsupported.		
	deals with the negative index soi	**B1**	e.g. $\left(\frac{3-\sqrt{5}}{1+\sqrt{5}}\right)^2$
	rationalises $\frac{3-\sqrt{5}}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$ oe	**M1**	allow for $\frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$
	multiplies out correctly $\frac{3-4\sqrt{5}+5}{1-5}$ oe	**A1**	allow for $\frac{3+4\sqrt{5}+5}{9-5}$
	squares correct binomial $\left(-2+\sqrt{5}\right)^2 = \left(4-4\sqrt{5}+5\right)$ oe	**A1**	allow for $\left(2+\sqrt{5}\right)^2 = \left(4+4\sqrt{5}+5\right)$
	$9-4\sqrt{5}$ cao	**A1**	dep on all previous marks awarded

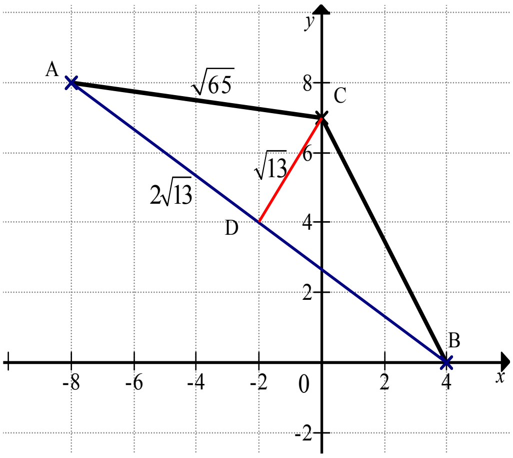
Question	Answer	Marks	Partial Marks
2	**Alternative method 1:** dealing with the negative index soi	B1	
	correctly squaring with at least 3 terms in the numerator and denominator $\frac{3-\sqrt{5}}{1+\sqrt{5}} \times \frac{3-\sqrt{5}}{1+\sqrt{5}} = \frac{9-6\sqrt{5}+5}{1+2\sqrt{5}+5}$ oe	B1	
	rationalising *their* $\left(\frac{14-6\sqrt{5}}{6+2\sqrt{5}} \times \frac{6-2\sqrt{5}}{6-2\sqrt{5}}\right)$ oe	M1	
	multiplying out correctly; at least 3 terms in the numerator but condone a single value for the denominator $\frac{84-64\sqrt{5}+60}{36-20}$ oe	A1	
	$9-4\sqrt{5}$ cao	A1	
	Alternative method 2 dealing with the negative index soi	B1	
	$9-6\sqrt{5}+5=\left(a+b\sqrt{5}\right)\left(1+2\sqrt{5}+5\right)$	M1	
	$14=6a+10b$ $-6=2a+6b$ oe	A1	
	$a=9$ cao	A1	
	$b=-4$ cao	A1	
	Alternative method 3 for dealing with the negative index soi	B1	
	[$3-\sqrt{5}=\left(c+d\sqrt{5}\right)\left(1+\sqrt{5}\right)$ leading to] $c+5d=3$ $c+d=-1$	M1	
	$c=-2$ and $d=1$	A1	
	$\left(-2+\sqrt{5}\right)^2=4-4\sqrt{5}+5$	A1	
	$9-4\sqrt{5}$ cao	A1	

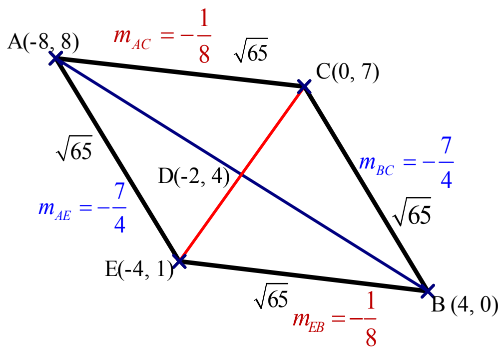
Question	Answer	Marks	Partial Marks
3	Correctly finding a correct linear factor or root	**B1**	from a valid method, e.g. factor theorem used or long division or synthetic division: $f(2)=10(2^3)-21(2^2)+4=0$ or $\begin{array}{r} 10x^2-x-2 \\ x-2\overline{)10x^3-21x^2\quad+4} \\ \underline{10x^3-20x^2} \\ -x^2 \\ \underline{-x^2+2x} \\ -2x+4 \\ \underline{-2x+4} \\ 0 \end{array}$ or $\begin{array}{c\|cccc} 2 & 10 & -21 & 0 & 4 \\ & \downarrow & 20 & -2 & -4 \\ \hline & 10 & -1 & -2 & 0 \end{array}$
	correct linear factor stated or implied by, e.g. $(x-2)(10x^2-x-2)$	**B1**	$(x-2)$ or $(2x-1)$ or $(5x+2)$ do not allow $\left(x-\frac{1}{2}\right)$ or $\left(x+\frac{2}{5}\right)$
	Correct quadratic factor $(10x^2-x-2)$ or $(5x^2-8x-4)$ or $(2x^2-5x+2)$	**B2**	found using any valid method; **B1** for any 2 terms correct
	$(x-2)(2x-1)(5x+2)$ mark final answer	**B1**	must be written as a correct product of all 3 linear factors; only award the final **B1** if **all** previous marks have been awarded
			If quadratic factor is not found but correct remaining linear factors are found using e.g. the factor theorem or long division or synthetic division etc. with correct, sufficient, complete working to justify that no calculator has been used allow: **B1** for correctly finding a correct linear factor or root **B1** for a correct linear factor stated or implied **SC3** for the full, complete and correct working to find the remaining two linear factors and arrive at the correct product of 3 linear factors

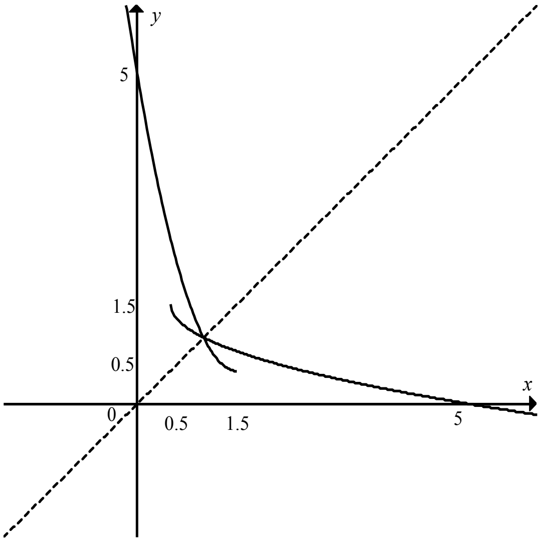
Question	Answer	Marks	Partial Marks
4	$\frac{dy}{dx} = 6x - 7$ soi	B1	
	$m_{\text{normal}} = -\frac{1}{5}$ soi	B1	finds or uses correct gradient of normal
	$m_{\text{tangent}} = 5$ soi or $(6x-7)\left(-\frac{1}{5}\right) = -1$ oe	M1	uses $m_1 m_2 = -1$ with numerical gradients
	$6x - 7 = 5$ oe $\Rightarrow x = 2$	A1	
	$y = 9$	A1	
	$k = 47$	A1	
	Alternative method $m_{\text{normal}} = -\frac{1}{5}$	B1	
	$m_{\text{tangent}} = 5$	M1	
	$3x^2 - 12x + 11 - c = 0$ oe	A1	
	solving $3x^2 - 12x + 12 = 0$ oe to find $x = 2$	A1	
	$y = 9$	A1	
	$k = 47$	A1	
5(i)	$\left(\textit{their}\, 2x^4\right)(0.2 - \ln 5x) + 0.4x^5\left(\textit{their}\, \frac{-5}{5x}\right)$ oe or $\textit{their}\, 0.4x^4 - \left(\left(\textit{their}\, 2x^4\right)\ln 5x + 0.4x^5\left(\textit{their}\, \frac{5}{5x}\right)\right)$ oe	M1	clearly applies correct form of product rule
	$-2x^4 \ln 5x$ isw	A1	nfww
5(ii)	$3\ln 5x$ or $\ln 5x + \ln 5x + \ln 5x$	B1	

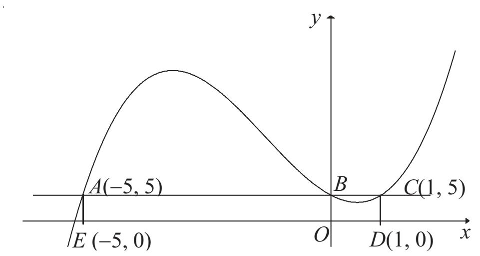
Question	Answer	Marks	Partial Marks
5(iii)	$\frac{3}{-2}\int\left(-2x^4\ln 5x\right)\mathrm{d}x$ oe	M1	**FT** $k = 2$ from (i) allow for $\frac{3}{2}\int\left(2x^4\ln 5x\right)\mathrm{d}x$ or, when $k = -2$, for $\int\left(x^4\ln 5x\right)\mathrm{d}x = -0.2x^5(0.2-\ln 5x)$ or $-\frac{2}{3}\int\left(3x^4\ln 5x\right)\mathrm{d}x = 0.4x^5(0.2-\ln 5x)$ oe or, when FT $k = 2$, for $\int\left(x^4\ln 5x\right)\mathrm{d}x = 0.2x^5(0.2-\ln 5x)$ or $\frac{2}{3}\int\left(3x^4\ln 5x\right)\mathrm{d}x = 0.4x^5(0.2-\ln 5x)$ oe
	$-\frac{3}{2}\left(0.4x^5(0.2-\ln 5x)\right)[+c]$ oe isw cao	A1	nfww; implies M1 An answer of $0.6x^5(0.2-\ln 5x)$ following $k = 2$ from (i) implies M1 A0
6	Uses b^2-4ac	M1	
	$(p-q)^2-4(p)(-q)$	A1	implies **M1**
	$p^2+2pq+q^2$	M1	correctly simplifies
	$(p+q)^2 \geqslant 0$ oe cao isw	A1	
	Alternative method $(px-q)(x+1)\quad[=0]$ or $\frac{-(p-q)\pm\sqrt{(p+q)^2}}{2p}$	M2	or **M1** for $(px+q)(x-1)\quad[=0]$ or $\frac{-(p-q)\pm\sqrt{(p-q)^2-4(p)(-q)}}{2p}$
	$x=\frac{q}{p},\quad x=-1$	A1	
	for conclusion, e.g. p and q are real therefore $\frac{q}{p}$ is real [and –1 is real]	A1	
7(a)(i)	7	B1	
7(a)(ii)	$\frac{1}{7}$ or $\frac{1}{their 7}$	B1	**FT** *their* 7 must not be 1 if following through

Question	Answer	Marks	Partial Marks
7(b)	$y=81^{-\frac{1}{4}}$ or $y=3^{-1}$ or $y=9^{-\frac{1}{2}}$ oe	**M1**	Anti-logs
	$y=\frac{1}{3}$ only or 0.333[3….] only	**A1**	nfww; implies the M1; $y=$ …. must be seen at least once If M0 then **SC1** for e.g. $81^{-\frac{1}{4}}=\frac{1}{3}$ as final answer
7(c)	$\frac{2^{5(x^2-1)}}{(2^2)^{x^2}}$ oe or $\frac{4^{\frac{5}{2}(x^2-1)}}{4^{x^2}}$ oe or $\frac{32^{x^2}\times 32^{-1}}{4^{x^2}}$ or $\log 32^{x^2-1}-\log 4^{x^2}=\log 16$ oe	**B1**	converts the terms given left hand side to powers of 2 or 4; may have cross-multiplied or separates the power in the numerator correctly or applies a correct log law
	$2^{3x^2-5}=16$ oe $\Rightarrow 3x^2-5=4$ oe or $4^{\frac{3}{2}x^2-\frac{5}{2}}=16$ oe $\Rightarrow \frac{3}{2}x^2-\frac{5}{2}=2$ oe or $\frac{8^{x^2}}{32}=16$ oe $\Rightarrow x^2\log 8=\log 512$ oe or $(x^2-1)\log 32-x^2\log 4=\log 16$ oe	**M1**	combines powers and takes logs or equates powers; or brings down all powers for an equation already in logs condone omission of necessary brackets for M1; condone one slip
	$[x=]\pm\sqrt{3}$ isw cao or ± 1.732050... rot to 3 or more figs. isw	**A1**	
8(i)	$y-8=-\frac{8}{12}(x-(-8))$ oe isw or $y[-0]=-\frac{8}{12}(x-4)$ oe isw or $3y=-2x+8$ oe isw	**B2**	**B1** for $m_{AB}=-\frac{8}{12}$ oe or **M1** for $\frac{8-0}{-8-4}$ oe
8(ii)	$(-8-4)^2+(8[-0])^2$ oe	**M1**	any valid method
	$\sqrt{208}$ isw or $4\sqrt{13}$ isw or 14.4222051… rot to 3 or more sf	**A1**	implies **M1** provided nfww

Question	Answer	Marks	Partial Marks
8(iii)	[coordinates of D =] $(-2, 4)$ soi	**B1**	If coordinates of D not stated then a calculation for m_{CD} or a relevant length with the coordinates clearly embedded must be shown to imply **B1**
	Gradient methods: $\left[m_{CD} = \frac{7 - their4}{0 - their(-2)} =\right]$ $their\begin{pmatrix}3\\2\end{pmatrix}$  	**M1**	or **Length of sides methods:** finds or states $AC^2 = 65$ or $AC = \sqrt{65}$ or $AC^2 = (-8-0)^2 + (8-7)^2$ oe or $AC = \sqrt{(-8-0)^2 + (8-7)^2}$ oe **and** $CD^2 = their13$ or $CD = their\sqrt{13}$ or $CD^2 = (0 - their(-2))^2 + (7 - their4)^2$ oe or $CD = \sqrt{(0 - their(-2))^2 + (7 - their4)^2}$ oe **and** $AD^2 = their52$ or $AD = their2\sqrt{13}$ or $AD^2 = (-8 - their(-2))^2 + (8 - their4)^2$ or $AD = \sqrt{(-8 - their(-2))^2 + (8 - their4)^2}$ or uses a valid method with *their* coordinates of D to find the exact area of the triangle and equates to $\frac{1}{2}(AD)(CD)\sin(ADC)$
	states $\frac{3}{2} \times \left(-\frac{8}{12}\right) = -1$ oe or $\frac{3}{2}$ is the negative reciprocal of $-\frac{2}{3}$ oe or finds the equation of the perpendicular bisector of AB as $y = \frac{3}{2}x + 7$ independently of C and states that C lies on this line.	**A1**	applies Pythagoras to confirm, using integer values, that 65 = 13 + 52 or finds e.g. $AC = \sqrt{65}$ using $\sqrt{(2\sqrt{13})^2 + (\sqrt{13})^2}$ or solves $\frac{1}{2}(2\sqrt{13})(\sqrt{13})\sin ADC = 13$ or $(\sqrt{65})^2 = (2\sqrt{13})^2 + (\sqrt{13})^2 - 2(2\sqrt{13})(\sqrt{13})\cos ADC$ to show ADC is a right angle
8(iv)	$\begin{pmatrix}-4\\1\end{pmatrix}$ or $-4\mathbf{i} + \mathbf{j}$	**B1**	condone coordinates

Question	Answer	Marks	Partial Marks
8(v)	Full valid method e.g. for **showing** that e.g. $\overrightarrow{CB} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}0\\7\end{pmatrix} = \begin{pmatrix}4\\-7\end{pmatrix}$ or **showing** that e.g. $\overrightarrow{AC} = \begin{pmatrix}0\\7\end{pmatrix} - \begin{pmatrix}-8\\8\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix}$ oe **and** $\overrightarrow{EB} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}-4\\-1\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix}$ oe or comparing gradients of both pairs of opposite sides and showing they are pairwise the same or comparing the lengths of both pairs of opposite sides and showing that they are pairwise the same or showing that length AC = length AE or that the length BC = length BE or comparing the gradients and lengths of a pair of opposite sides or showing that D is the midpoint of CE or showing that length DC = length DE and that C, D and E are collinear	B2	**B1** for incomplete method e.g. for **stating** that $\overrightarrow{CB} = \begin{pmatrix}4\\-7\end{pmatrix}$ or $\overrightarrow{AC} = \begin{pmatrix}8\\-1\end{pmatrix} = \overrightarrow{EB}$ or just showing that one pair of opposite sides is parallel or has the same length or just showing that length DC = length DE or just showing that C, D and E are collinear 
9(i)	$2(x-1.5)^2 + 0.5$ isw	B3	or **B3** for $a = 2$ and $b = 1.5$ and $c = 0.5$ provided not from wrong format isw or **B2** for $2(x-1.5)^2 + c$ where $c \neq 0.5$ or $a = 2$ and $b = 1.5$ or **SC2** for $2(x-1.5)+0.5$ or $2\left((x-1.5)^2 + \frac{1}{4}\right)$ seen or **B1** for $(x-1.5)^2$ seen or for $b = 1.5$ or for $c = 0.5$ or **SC1** for 3 correct values seen in incorrect format e.g. $2(x-1.5x)+0.5$ or $2(x^2-1.5)+0.5$

Question	Answer	Marks	Partial Marks
9(ii)		**B3**	**B1** for correct graph for f over correct domain or correct graph for f – 1 over correct domain **B1** for vertex marked for f or f – 1 and intercept marked for f or f – 1 **B1** for idea of symmetry – either symmetrical by eye, ignoring any scale or line $y = x$ drawn and labelled Maximum of 2 marks if not fully correct
9(iii)	$\frac{x-0.5}{2}=(y-1.5)^2$	**M1**	**FT** *their a,b,c*, provided *their a* $\neq 1$ and a,b,c are all non-zero constants or $\frac{y-0.5}{2}=(x-1.5)^2$ and reverses variables at some point
	$\mathrm{f}^{-1}(x)=1.5-\sqrt{\frac{x-0.5}{2}}$ oe	**A1**	must have selected negative square root only; condone $y = ...$ etc.; must be in terms of x
			If M0 then **SC2** for $\mathrm{f}^{-1}(x)=\frac{6-\sqrt{8x-4}}{4}$ oe or **SC1** for $\mathrm{f}^{-1}(x)=\frac{-(-6)\pm\sqrt{36-4(2)(5-x)}}{2(2)}$ oe
	$x \geqslant \frac{1}{2}$ oe	**B1**	
10(i)	$\sin^{-1}\left(\frac{3}{4}\right)$ soi	**M1**	implied by 0.848[06…]
	0.848[06…] rot to 3 or more figs or 2.29[35…] rot to 3 or more figs	**M1**	implied by a correct answer of acceptable accuracy
	0.544 486... rot to 3 or more figs isw	**A1**	
	1.03 or 1.02630... rot to 4 or more figs isw	**A1**	Maximum 3 marks if extra angles in range; no penalty for extra values outside range $0 \leqslant x \leqslant \frac{\pi}{2}$

Question	Answer	Marks	Partial Marks
10(ii)	Correctly uses $\tan^2 y = \sec^2 y - 1$ and/or $\frac{\sin y}{\cos y}$ **and** $\sin^2 y = 1 - \cos^2 y$	**M1**	for using correct relationship(s) to find an equation in terms of a single trigonometric ratio
	$3\sec^2 y - 14\sec y - 5 = 0$ $\Rightarrow (3\sec y + 1)(\sec y - 5)$ or $5\cos^2 y + 14\cos y - 3 = 0$ $\Rightarrow (5\cos y - 1)(\cos y + 3)$	**DM1**	for factorising or solving their 3-term quadratic dependent on the first M1 being awarded
	$[\cos y = -3]$ $\cos y = \frac{1}{5}$	**A1**	
	78.5 or 78.4630... rot to 2 or more decimal places isw	**A1**	
	281.5 or 281.536.... rot to 2 or more decimal places isw	**A1**	Maximum 4 marks if extra angles in range; no penalty for extra values outside range $0 \leqslant x \leqslant 360$
11(i)	$\frac{x^4}{4} + \frac{4x^3}{3} - \frac{5x^2}{2} + 5x\,[+c]$ isw	**B2**	**B1** for any 3 correct terms
11(ii)	$x^3 + 4x^2 - 5x + 5 = 5$ and rearrange to $x(x^2 + 4x - 5) = 0$ oe soi	**B1**	
	Solves *their* $x^2 + 4x - 5\,[= 0]$ soi	**M1**	
	$x = -5,\ x = 1$ soi	**A1**	
	$OEAB = 25,\ OBCD = 5$	**A1**	

Question	Answer	Marks	Partial Marks
11(iii)	Correct or correct **FT** substitution of 0, *their* –5 seen in $\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x\right]_{their-5}^{0}$	**M1**	dependent on at least **B1** in (i)
	Correct or correct **FT** substitution of *their* 1, 0 seen in $\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x\right]_{0}^{their1}$	**M1**	dependent on at least **B1** in (i)
	$their\frac{1175}{12}-theirOEAB+theirOBCD-their\frac{49}{12}$ oe	**M1**	for the strategy needed to combine the areas; may be in steps; $97.91\dot{6}-25+5-4.08\dot{3}$
	$\frac{886}{12}$ oe or $73\frac{5}{6}$ oe or $73.8\dot{3}$ rot to 3 or more sig figs	**A1**	all method steps must be seen; not from wrong working If M0 then allow **SC3** for $\int_{-5}^{0}(x^3+4x^2-5x)\,dx-\int_{0}^{1}(x^3+4x^2-5x)\,dx$ oe $=\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{-5}^{0}-\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{0}^{1}$ $=\left[0-\left(\frac{625}{4}-\frac{500}{3}-\frac{125}{2}\right)\right]-\left[\left(\frac{1}{4}+\frac{4}{3}-\frac{5}{2}\right)-0\right]$ $=\frac{443}{6}$ oe or **SC2** for $\int_{their(-5)}^{0}(x^3+4x^2-5x)\,dx-\int_{0}^{their1}(x^3+4x^2-5x)\,dx$ oe $=\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{their(-5)}^{0}-\left[\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}\right]_{0}^{their1}$ $=[F(0)-F(their(-5))]-[F(their1)-F(0)]$
12(i)	$-6(2x+1)^{-2}$ or $\frac{-6}{(2x+1)^2}$ oe isw	**B1**	Allow $-3(2x+1)^{-2}\times 2$ or $\frac{-3\times 2}{(2x+1)^2}$ oe
	Denominator or $(2x+1)^2$ is positive [and numerator negative therefore $g'(x)$ is always negative] oe	**B1**	**FT** *their* $g'(x)$ of the form $\frac{-k}{(2x+1)^2}$ oe where $k>0$; Allow $(2x+1)^{-2}$ is always positive
12(ii)	$g>0$	**B1**	
12(iii)	$\frac{3k}{2x+1}+3$ oe isw	**B1**	

Question	Answer	Marks	Partial Marks
12(iv)	$\frac{3k}{2(0)+1}+3=5$	**B1**	
	$k=\frac{2}{3}$ isw	**B1**	implies the first **B1**
12(v)	$x>-\frac{1}{2}$	**B1**	